

Qudit-Teleportation for photons with linear optics

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Quantum Teleportation, the transfer of the state of one quantum system to another without direct interaction between both systems, is an important way to transmit information encoded in quantum states and to generate quantum correlations (entanglement) between remote quantum systems. So far, for photons only superpositions of two distinguishable states (one "qubit") could be teleported. We here show how to teleport a "qudit", i.e., a superposition of an arbitrary number d of distinguishable states present in the orbital angular momentum of a single photon using d beam splitters and d additional entangled photons. The same entanglement resource might also be employed to collectively teleport the state of $d/2$ photons at the cost of one additional entangled photon per qubit. This is superior to existing schemes for qubits which require an additional pair of entangled photons per qubit.

In classical physics it is possible, in principle, to detect the state of a single system, for example the position and momentum of a point particle, transmit the information about that state to a remote location and then reconstruct it within a second system. This concept of "classical teleportation" underlies telecommunication techniques such as the transfer of documents via facsimile. Quantum physics, however, excludes the possibility to detect or duplicate the state of a single microscopic system¹ and therefore rules out all forms of classical teleportation with atoms, photons or other quantum systems. It is thus surprising², that the state transfer between quantum systems can nevertheless be realized according to the rules of quantum physics by means of "quantum teleportation"³. This procedure makes use of correlations between quantum systems - entanglement - which cannot be described by local-realistic theories⁴, such as classical mechanics or electrodynamics nor any other theory within classical physics.

Quantum teleportation lies at the core of quantum communication which is the quantum analog of telecommunication, and can also be employed to enhance the success probability in quantum computing with photons⁵⁻⁸. Moreover, it is one of the crucial ingredients^{9,10} for enabling long-distance quantum cryptography - a technique to transmit information secured against eavesdropping.

The importance of quantum teleportation for quantum information processing and communication can be seen from the long list of experimental realizations of teleportation of the state of a two-level system corresponding to the smallest unit of quantum information - one quantum bit (qubit),¹¹⁻²³. In these realizations single qubits were encoded in the polarization of photons or in the superposition of vacuum and one photon states²¹. Quantum teleportation with two-level atoms has been demonstrated²⁴⁻²⁶ and it has also been designed for three- and four-level atomic systems^{27,28}.

At present, light is the only candidate for quantum communication and quantum cryptography over large distances because of its small interaction with its envi-

ronment as compared to matter²⁹. At the same time its small interaction makes it difficult to manipulate the states of light in order to achieve teleportation with photons. There are two main challenges in realizing teleportation: (i) photons sharing maximal quantum correlations (entanglement) have to be generated and distributed between the sender and the receiver of quantum information and (ii) the input photons and the photons of the sender have to be projected into a maximally entangled state by a joint measurement of both (a so-called Bell measurement) in order to transfer the state of the input photons to the photons held by the receiver. Both challenges can in principle be overcome using non-linear optical media, which manipulate the light depending on its intensity. The corresponding processes, however, have a very small efficiency on the *single photon level*. For example a non-linear Beta Barium Borate (BBO) crystal is used to generate, from one photon, two photons of half the input-frequency in an entangled state (challenge (i)) with a success probability of approximately 10^{-6} per incoming photon³⁰. The efficiency of a Bell measurement by means of non-linear optics in order to meet challenge (ii) is even lower¹⁷, at about 10^{-10} . Therefore it is desirable to design a more efficient solution to both challenges based on linear optics. Here we focus on challenge (ii) and briefly discuss a solution of challenge (i) which will be given explicitly in a subsequent article.

Although linear optics, that is, the use of beam splitters, phase shifters and mirrors, does not allow the realization of a complete Bell measurement³¹, a simple 50 : 50 beam splitter can be used as a filter to project two incoming photons onto a particular entangled state in a certain percentage of the cases. Two photons incident on the input ports of a beam splitter do not produce a coincidence count in two detectors in the output ports (Hong-Ou-Mandel effect^{32,33}) unless they possess an anti-symmetric component with respect to their internal degree of freedom, e.g. their polarization. A coincidence count thus effectively projects onto an antisymmetric state. For two polarized photons entering in dif-

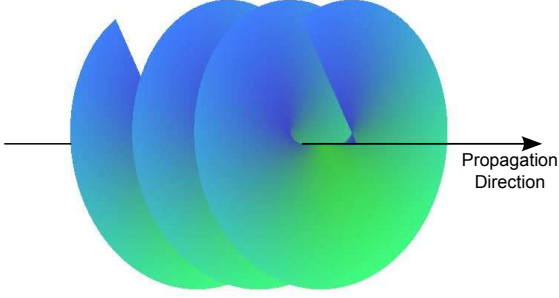


FIG. 1: Schematic diagram of the helical wavefront of a light beam.

ferent input ports of the beam splitter there is only one such state:

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|HV\rangle - |VH\rangle), \quad (1)$$

i.e, elementary excitations of the first and second spatial mode (represented by the first and second slot in the state symbol) which carry horizontal and vertical polarization, respectively, superposed with excitations of these modes with swapped polarizations. This state is antisymmetric, because it changes sign under a permutation of the first and second mode (slot), and maximally entangled, a condition that allows the realization of the teleportation of a qubit¹¹ encoded in the polarization of a single photon. Moreover, as we shall see in the following, this phenomenon can also be employed for the simultaneous teleportation of multiple qubits encoded in the orbital angular momentum (OAM) of photons.

It was noticed by Allen et al.³⁴ in 1992 that light with a phase distribution $\exp(il\phi)$ depending on the azimuthal angle ϕ in the plane orthogonal to its direction of propagation carries an orbital angular momentum of an integer l times Planck's constant \hbar per photon. Such light is characterized by helical (screw-like) wavefronts (cp. Fig. 1) and can for example be produced by means of spatial light modulators - thin liquid crystal displays (LCDs) which imprint the helical phase pattern or superpositions of such patterns. The orbital angular momentum of a photon can thus be used to carry information and represents a quantum system with an unrestricted number of levels.

Quantum teleportation using an incomplete Bell measurement (a “Bell filter”) can be applied to systems with an arbitrary number of levels, not only simple two-level systems (such as polarized photons). Let us review how. Teleportation involves three parties Alice, Bob and Charlie, cp. Fig 2. Alice and Bob are far apart and share a

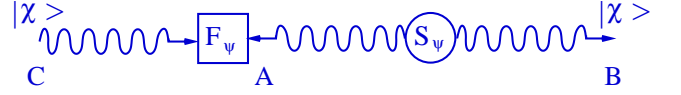


FIG. 2: Teleportation using a filter: source S_ψ produces a pair of systems A and B in state $|\Psi\rangle$. System A and system C are to be projected by a filter F_ψ into state $|\Psi\rangle$. If successful, the filtering transfers the initial state of C to system B .

pair of systems in a maximally entangled state

$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{D}} \sum_{i=0}^{D-1} |A_i\rangle_A \otimes |B_i\rangle_B, \quad (2)$$

which is a superposition of products of orthogonal basis states of their systems A and B . Charlie provides Alice with an unknown state $|\chi\rangle$, which has to be transferred from Charlie's system C to Bob's system B . For this purpose systems C and B must be similar - B has to support the same states as C . The state to be teleported can thus be expressed as a superposition of Bob's basis states: $|\chi\rangle = \sum_{k=0}^{D-1} \alpha_k |B_k\rangle$. Alice successfully teleports the state $|\chi\rangle$ if she carries out a measurement which acts like a filter and projects systems C and A onto the entangled state $|\Psi\rangle$:

$$\begin{aligned} |\chi\rangle_C \otimes |\Psi\rangle_{AB} &\rightarrow (|\Psi\rangle\langle\Psi|)_{CA} \otimes \mathbf{1}_B |\chi\rangle_C \otimes |\Psi\rangle_{AB} \\ &= \frac{1}{D^{3/2}} \sum_{k,l=0}^{D-1} \alpha_k |B_l\rangle_C \otimes |A_l\rangle_A \otimes |B_k\rangle_B \\ &= \frac{1}{D} |\Psi\rangle_{CA} \otimes |\chi\rangle_B. \end{aligned} \quad (3)$$

According to the rules of quantum mechanics the likelihood for such a projection to occur is given by the square of the length of the resulting state vector, $p = 1/D^2$. Thus the success probability of this teleportation scheme, which uses only one of the outcomes of a Bell measurement, decreases with the number D of basis states in which quantum information is encoded. The advantage lies in the fact that this concept of teleportation, which is used for photonic qubits ($D = 2$) can be generalised to teleport states with arbitrary D using linear optics. In order to do so, we have to identify a photonic system with a unique antisymmetric state and a linear optical device that plays the role of the beam splitter in the qubit case, i.e., a filter for antisymmetric states. The uniqueness is required to guarantee that the filter yields the same state $|\Psi\rangle_{CA}$ which is initially shared by Alice and Bob, i.e, the state $|\Psi\rangle_{AB}$. The dimension of the space spanned by the antisymmetric states of composite systems (only if the dimension equals one do we have a unique antisymmetric state!) can be easily determined by means of Young tableaux (see Supplementary Information). It turns out that only d systems each with d levels possess a unique antisymmetric state. As a consequence, a generalization of the teleportation scheme for photonic qubits by means

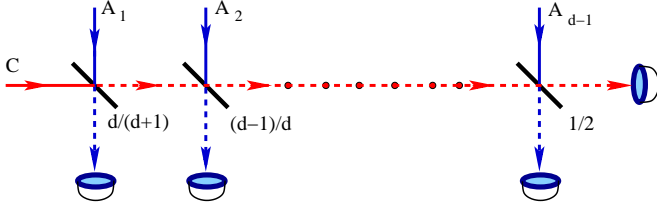


FIG. 3: Bell filter: array of d beam splitters which projects Charlie's incoming photon and Alice's $d - 1$ photons into the antisymmetric state $|\Psi\rangle$. Below each beamsplitter its reflectivity is indicated.

of a Bell filter for antisymmetric states requires d photons propagating on different paths each with a quantized degree of freedom, e.g. OAM, with d -levels, i.e., d qudits.

Photonic states can be conveniently expressed by means of creation operators a^\dagger acting on the vacuum state $|0\rangle$. In our case these operators carry two indices — the first one, j , specifies one of d possible propagation paths whereas the second one, $l = 1 \dots d$, denotes the orbital angular momentum value $l\hbar$ of the photon. For example the state $|\psi\rangle = |12\rangle - |21\rangle$ of two photons propagating on different paths with two OAM values $l = 1, 2$ — which is the OAM analog of the polarization state in Eq. (1) — can be written by means of a determinant of creation operators:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} a_{11}^\dagger a_{22}^\dagger - a_{12}^\dagger a_{21}^\dagger \end{pmatrix} |0\rangle = \frac{1}{\sqrt{2}} \det \begin{pmatrix} a_{11}^\dagger & a_{12}^\dagger \\ a_{21}^\dagger & a_{22}^\dagger \end{pmatrix} |0\rangle \quad (4)$$

It is obvious that the state $|\psi\rangle$ is antisymmetric, since under permutation of the propagation paths it is transferred to $-|\psi\rangle$. The antisymmetry is represented by the determinant: a swap of rows corresponding to the permutation results in a minus sign of the determinant. Using the same logic the antisymmetric state of d photons with d OAM values can be expressed by the determinant of a $d \times d$ Matrix Λ

$$|\Psi\rangle = \frac{1}{\sqrt{d!}} \det(\Lambda) |0\rangle \quad (5)$$

where

$$\Lambda = \begin{pmatrix} a_{11}^\dagger & a_{12}^\dagger & \cdots & a_{1d}^\dagger \\ a_{21}^\dagger & a_{22}^\dagger & \cdots & a_{2d}^\dagger \\ \vdots & \vdots & \ddots & \vdots \\ a_{d1}^\dagger & a_{d2}^\dagger & \cdots & a_{dd}^\dagger \end{pmatrix} \quad (6)$$

Here $|\Psi\rangle$ is the antisymmetric state which was sought, since a permutation of any two propagation directions corresponding to a swap of two rows of the determinant in Eq. (5) leads to a change of sign of the state $|\Psi\rangle$. It turns out that a certain array of d beam splitters (cp. Fig. 3) — acts as a Bell filter and projects onto $|\Psi\rangle$ in case of a coincidence count at all output ports, which can be

checked for any finite dimension d by direct calculation. Our notation makes it a simple matter to understand the reason. Since the beam splitters only superimpose the incoming light fields (first index of the creation operators) independent of their orbital angular momenta (second index), the resulting linear transformation can be expressed by a rotation matrix U acting on the creation operators

$$\tilde{a}_{kj}^\dagger = \sum U_{ki} a_{ij}^\dagger, \quad (7)$$

which leads to a linear transformation of the matrix Λ

$$\tilde{\Lambda} = U \Lambda \quad (8)$$

and the invariance of the antisymmetric state: $|\Psi\rangle \rightarrow \det(\tilde{\Lambda})|0\rangle = \det(U) \det(\Lambda)|0\rangle = \det(\Lambda)|0\rangle = |\Psi\rangle$. Hence one photon will leave from each of the d output ports of the multiport thus leading to a possible coincidence count.

In order to use the antisymmetric state given in Eq. (5), the d photons must be divided between Alice (n photons) and Bob ($d - n$ photons) such that both share a bipartite maximally entangled state. If Charlie now provides Alice with $d - n$ photons, she can teleport the state of these photons by projecting them onto an antisymmetric state by means of a balanced multiport. In order to check for maximal entanglement we once more take advantage of the representation of $|\Psi\rangle$ in terms of a determinant. Expanding the determinant with respect to the first row, we obtain the state:

$$\begin{aligned} |\Psi\rangle &= \frac{1}{\sqrt{d!}} \det(\Lambda) |0\rangle = \frac{1}{\sqrt{d!}} \sum_{i=1}^d (-1)^{i+1} a_{1i}^\dagger \det(\Lambda_{1i}) |0\rangle \\ &= \frac{1}{\sqrt{d}} \sum_{i=1}^d |A_i\rangle |i\rangle, \end{aligned} \quad (9)$$

where $|A_i\rangle = 1/\sqrt{(d-1)!} \det(\Lambda_{1i}) |0\rangle$ and Λ_{1i} is the $(d-1) \times (d-1)$ submatrix obtained by omitting the i -th column and the first row (a so-called minor of Λ) and $(-1)^{i+1} a_{1i}^\dagger |0\rangle = |i\rangle$. It is remarkable that the expansion of the determinant results in a maximally entangled bipartite state of the form (2) with $D = d$ and $|B_i\rangle = |i\rangle$, implying that Alice and Bob obtain $d - 1$ and one photon, respectively. This partition of photons allows Alice to teleport any state of a single d -level photon from Charlie by sending it along with her $d - 1$ photons into a balanced multiport and subsequently obtaining a coincidence count in all its output ports.

But this is not the only possible partitioning of photons which leads to a maximally entangled state between Alice and Bob. Strikingly, any partition $(n, d - n)$ with $0 < n < d$ of an antisymmetric state of d particles possesses this property, and this can be easily understood by virtue of the rules to calculate determinants (see Supplementary Information). For the partition $(n, d - n)$ one obtains a bipartite state as given in Eq. (2) with $D = \binom{d}{n}$ and $|A_i\rangle$ as well as $|B_i\rangle$ given in terms of minors of Λ . The

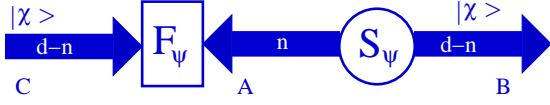


FIG. 4: Teleportation of any antisymmetric state $|\chi\rangle$ of $d-n$ photons: source S_ψ produces a pair of systems A and B in state $|\Psi\rangle$ with n and $d-n$ photons respectively. System A with n photons and system C with $d-n$ photons are to be projected by a filter F_ψ into state $|\Psi\rangle$. If successful, the filtering transfers the initial state $|\chi\rangle$ of C to system B .

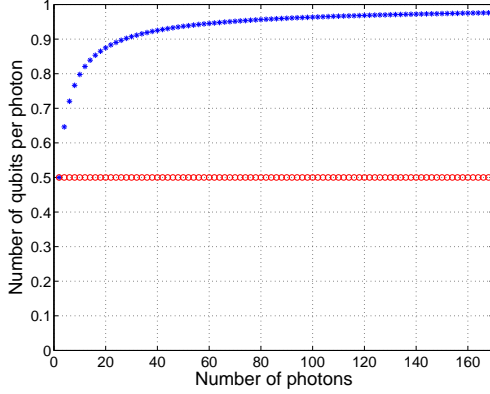


FIG. 5: The graphs show the number of quantum information units (qubits) teleported per additional photon versus the number of additional photons required for individual-qubit teleportation (red) and the optimal new technique (blue).

maximum amount of quantum information can be teleported with a $(d/2, d/2)$ partition of an even number d of photons prepared in state $|\Psi\rangle$. In this case Charlie has to provide $d/2$ photons, cp. Fig. 4, and can send for large d simultaneously approximately d qubits encoded in the $D = \binom{d}{d/2} = 2^d + O(d)$ dimensional, anti-symmetric subspace spanned by the $|B_i\rangle$. This is an exponential gain compared to the teleportation of a single qudit (corresponding to $\log_2(d)$ qubits) carried by one photon, which requires the same entanglement resource – an antisymmetric state of d photons $|\Psi\rangle$.

In comparison, teleporting d qubits individually requires the same number of maximally entangled photon pairs, i.e., a total of $2d$ additional photons, resulting in an efficiency of sending half a qubit per additional photon or $1/3$ of a qubit per photon generated. Our scheme yields double these rates: it thus requires half the number photons (cp Fig. (5)) to teleport the same amount of quantum information. This is an important improvement since the number of photons, that can be generated per time-unit is the limiting factor for the bandwidth of photonic quantum communication.

In the foregoing discussion we assumed the existence of a source S_Ψ that produces the state $|\Psi\rangle$ initially shared by Alice and Bob (cp. challenge (i)). Indeed for qutrits, such a source can be realized by yet another balanced

multiport which acts as a Bell-filter for the antisymmetric state $|\Psi\rangle$, provided the input consist of a single photon and a antisymmetric state of two photons and one photon leaves each of its output ports. In the general case nondestructive heralding techniques based on non-linear optical effects^{38,39} might be employed, which however reduce the efficiency of the teleportation scheme and will be discussed together with other preparation methods elsewhere.

Supplementary Information

Dimension of the antisymmetric subspace

The dimension of an antisymmetric subspace can be calculated using combinatorial objects called Young tableaux, which provide a technique of keeping track of the constraints imposed by the permutation symmetry of the system. Here we represent a basis state of a system by a box, \boxed{a} , where a numbers the basis state. A basis of the symmetric combinations of two systems can be depicted by a row of two boxes, $\boxed{}\boxed{}$. Similarly, a basis of anti-symmetric states is represented by a column of boxes, $\begin{bmatrix} \\ \end{bmatrix}$. Since we are interested only in the antisymmetric part, we focus on columns only. The dimension of the corresponding subspace, i.e., the number of basis states, is obtained by counting the possibilities for filling the boxes with numbers according to certain rules. For the antisymmetric subspace we start filling the numbers in descending order, from top to bottom. For a system which consists of two subsystems, a Young tableaux reads:

$$\begin{bmatrix} \boxed{a} \\ \boxed{b} \end{bmatrix} \equiv |ab\rangle - |ba\rangle, \quad (10)$$

where a is always greater than b . Therefore, if the total number of basis states available for each subsystem is two, i.e., we are dealing with two qubits, there is only one possibility, namely $a = 2$ and $b = 1$, therefore we obtain an antisymmetric subspace of dimension one. If the available states are more than two, say d , then we have $d-1$ options for a and given a , $a-1$ options for b . As a result the total number of combinations is given by $1+2+\dots+d-1 = d(d-1)/2$, which is the dimension of the antisymmetric subspace for a pair of d -level systems each of which can carry one "qudit" of quantum information.

This can be generalized for systems with n subsystems. Now we have n numbers $\{a_1 > a_2 > \dots > a_n\}$ in a column of boxes. The dimension of this antisymmetric subspace is given by the binomial coefficient $\binom{d}{n}$, i.e, d choose n , where n is the number of subsystems. This equals one only when $n = d$, giving us a unique antisymmetric state for d qudits.

Laplace expansion for the determinant of a matrix

The determinant of an $n \times n$ matrix A with elements a_{ij} can be calculated by an expansion with respect to the

first row of A as follows:

$$\det(A) = \sum_{i=1}^n (-1)^{1+i} a_{1i} \det(A_{1i}). \quad (11)$$

Here $\det(A_{1i})$ is the determinant of the $(n-1) \times (n-1)$ submatrix of A obtained from eliminating the first row and i -th column. In fact this is just a special case of a simultaneous expansion of the determinant with respect to several rows. For example, expanding with respect to the first two rows of a 4×4 matrix A , we obtain:

$$\det(A) = \begin{array}{c} \text{[Grid 1]} - \text{[Grid 2]} + \text{[Grid 3]} \\ + \text{[Grid 4]} - \text{[Grid 5]} + \text{[Grid 6]} \end{array},$$

where each block on the right-hand side represents the product of the determinant of the submatrix (minor) with blue dot and the minor with the red dot. In general, any such Laplace expansion assumes the form $\det(A) = \sum_i c_i \det(A_i) \det(B_i)$, where $c_i = \pm 1$ and the A_i (B_i) are minors of A which differ at least in one column⁴⁰. The possible Laplace expansions of $\det \Lambda$ in (5) correspond to the different distributions of the d photons between Alice and Bob. Each distribution leads to orthogonal states $|A_i\rangle \propto \det(A_i)|0\rangle$ on Alice's side, and on Bob's side accordingly, and therefore to a maximally entangled state shared between both parties.

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